

CHAPTER 2

PRELIMINARY DEFINITIONS AND RELATIONS

يتكون حجم معين من التربة من حبيبات صلبة وهواء وماء (مياه جوفية أو مياه رشح). ومصدر القوة في التربة هو الحبيبات الصلبة. أما الهواء والمياه الذان يملآن الفراغات المتصلة بين الحبيبات فليس لهما أى مقاومة. لذلك فإن تحديد نسبة الحبيبات الصلبة في التربة وكافة العلاقات الحجمية والوزنية بين مكونات التربة الثلاثة (الحبيبات – الماء – الهواء) يعتبر أساسا للتعامل مع التربة حسابيا وهندسيا.

2.1 Soil as three phase system

Soil mass a three phase system consisting of:

- (a) solid particles (grains)
- (b) water
- (c) air

The voids space between soil particles is filled partly with water and partly with air, Fig 2.1 (a). In case of fully saturated soil, the voids are completely with water. The three soil constituents are mixed together forming certain soil structure. For calculation purpose, it is always more convenient to show these constituents in separate spaces, as shown in Fig. 2.1 (b). The total volume of soil (V) consists of :

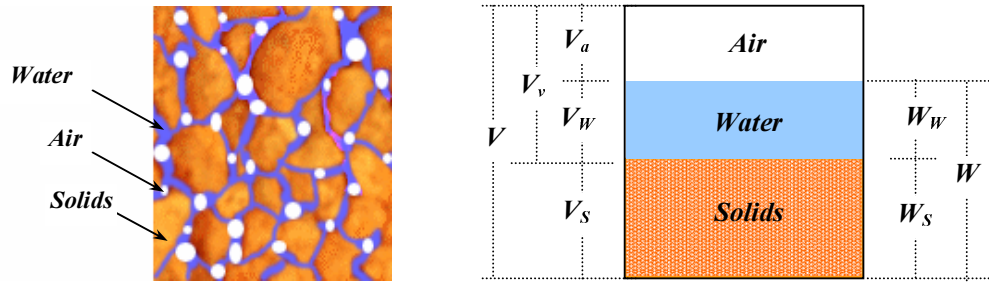


Fig. 2.1 Substitutions for phase calculations

- (a) Volume of air (V_a)
- (b) Volume of water (V_w)
- (c) Volume of solids (V_s)

The total volume of voids (V_v) is given by:

$$V_v = V_a + V_w$$

The total weight of soil (W) consists of:

- (a) Weight of water (W_w)
- (b) Weight of solids (W_s).

2.2 Definitions

2.21 water content: (w) محتوى الرطوبة

The water content (also called the moisture content) is defined as the ratio between the weight of water (W_w) and the weight of solids (W_s) in a given mass, i.e:

$$w = \frac{W_w}{W_s} \quad \dots \dots \dots (2.1)$$

The water content is usually expressed in percentage:

$$w (\%) = \frac{W_w}{W_s} \times 100 \quad \dots \dots \dots (2.2)$$

2.2.2 Unit weight of solid mass: (Density of soil)

(a) Bulk Density: (γ_b) الكثافة الكلية

The bulk density of soil is defined as the total weight of soil mass (W) per unit of its total volume (V), i.e:

$$\gamma_b = \frac{W}{V} \quad \dots \dots \dots (2.3)$$

(b) Dry Density: (γ_d) الكثافة الجافة

The dry density is defined as the weight of solids (W_s) per unit of total volume (V), i.e:

$$\gamma_d = \frac{W_s}{V} \quad \dots \dots \dots (2.4)$$

(c) Saturated Density: (γ_{sat}) الكثافة المشبعة

When the soil is fully saturated with water, its bulk density is called saturated density. Thus it is defined as the total weight of a saturated soil mass per unit of its total volume, i.e:

$$\gamma_{sat} = \frac{W_{sat.}}{V} \quad \dots \dots \dots (2.5)$$

(d) Submerged Density: ($\gamma_{sub.}$) الكثافة المغمورة

When a soil mass is submerged under water its weight is reduced due to bouyancy. The upward force is equal to the volume multiplied by the density of water (γ_w):

$$\gamma_{sub} = \frac{W - V \times \gamma_w}{V} = \gamma_{sat.} - \gamma_w \quad \dots \dots \dots (2.6)$$

For $\gamma_w = 1 \text{ gm / cm}^3$ (or 1 t/m^3):

$$\gamma_{\text{sub.}} = \gamma_{\text{sat.}} - 1 \quad \dots \dots \dots (2.7)$$

The density of soil is generally expressed in gm/cm^3 or t/m^3 units. It is considered as bulk density (also called unsaturated density) unless otherwise stated. Natural density is the bulk density of natural soil. It is generally taken as bulk density unless the soil is saturated or dry.

2.2.3 Specific gravity: (G_s) الوزن النوعي

The specific gravity of soil is defined as the ratio of the unit weight of soil solids to that of water, i.e:

$$G_s = \frac{W_s}{V_s} \cdot \frac{1}{\gamma_w} \quad \dots \dots \dots (2.8)$$

The specific gravity is therefore a dimensionless value.

2.2.4 Void ratio: (e) نسبة الفراغات

The void ratio is defined as the ratio of the volume of voids to the volume of soil solids, i.e. :

$$e = \frac{V_v}{V_s} \quad \dots \dots \dots (2.9)$$

2.2.5 Porosity: (n) المسامية

The porosity is defined as the ratio of the volume of voids to the total volume of soil, i.e. :

$$n = \frac{V_v}{V} \quad \dots \dots \dots (2.10)$$

The relationship between the porosity and void ratio is :

$$n = \frac{e}{1 + e} \quad \dots \dots \dots (2.11)$$

Also:

$$e = \frac{n}{1 - n} \quad \dots \dots \dots (2.12)$$

2.2.6 Degree of saturation: (S) درجة التشبع

The volume of voids in a soil mass is generally occupied by water and air. The ratio between the volume of water and the volume of voids is called the degree of saturation, i.e. :

$$S = \frac{V_w}{V_v} \quad \dots \dots \dots (2.13)$$

The degree of saturation is also expressed as percentage, i.e. :

$$S (\%) = \frac{V_w}{V_v} \times 100 \quad \dots \dots \dots (2.14)$$

In a saturated soil where the voids are totally filled with water the degree of saturation $S = 1$ (or 100%). For dry soil $S = 0$.

2.3 Functional relations

Relations between soil parameters defined in Section 2.2 are given as follows :

$$e \times S = G_s \times w \quad \dots \dots \dots (2.15)$$

$$\gamma_b = \frac{G_s + e.S}{1 + e} \cdot \gamma_w \quad \dots \dots \dots (2.16)$$

$$\gamma_d = \frac{G_s}{1 + e} \cdot \gamma_w \quad \dots \dots \dots (2.17)$$

$$\gamma_{sat.} = \frac{G_s + e}{1 + e} \cdot \gamma_w \quad \dots \dots \dots (2.18)$$

$$\gamma_{sub.} = \frac{G_s - 1}{1 + e} \cdot \gamma_w \quad \dots \dots \dots (2.19)$$

$$\gamma_b = \gamma_d (1 + w) \quad \dots \dots \dots (2.20)$$

جدول (٢-١)
قيم تقديرية لنسب مكونات بعض أنواع من التربة المصرية

نوع التربة	محتوى الرطوبة (%)	الكثافة الكلية طن/م ^٣	الكثافة الجافة طن/م ^٣	الكثافة المشبعة طن/م ^٣	الوزن النوعي	نسبة الفراغات	درجة التشبع (%)
تربة نيلية طينية (المهندسين)	٣٤	١.٨١	١.٣٥	١.٨٥	٢.٧	٠.٩٩٨	٩٢
تربة نيلية طميية (القناطر)	٤٢	١.٧٤	١.٣٣	١.٧٧	٢.٦٨	١.١٨	٩٥
تربة رملية (مدينة نصر)	٦	١.٨٠	١.٧٠	—	٢.٦٥	٠.٥٦	٢٨
تربة طينية انتفاشية (١٠ رمضان)	١٠	١.٩٩	١.٨١	—	٢.٧٢	٠.٥٠	٥٤
تربة طينية بحرية (المتزلة)	٥٤	١.٦٦	١.٠٨	١.٦٨	٢.٦٧	١.٤٨	٩٨

2.4 Examples

(1) The weight of a partially saturated soil sample is 600 gm and its volume is 365 cm^3 . After oven drying the weight of the sample reduces to 543 gm. Taking the specific gravity 2.67, find the water content, void ratio and degree of saturation. If the sample is saturated with water without change of its volume, find the saturated density.

Solution:

$$w = \frac{600 - 543}{543} = 0.105 = 10.5 \%$$

$$\therefore \gamma_b = \gamma_d(1 + w)$$

$$\therefore \gamma_b = \frac{600}{365} = 1.644 \text{ gm/cm}^3$$

$$1.644 = \gamma_d(1 + 0.105)$$

$$\gamma_d = 1.488 \text{ gm/cm}^3$$

$$\therefore \gamma_d = \frac{G_s}{1 + e} \gamma_w$$

$$1.488 = \frac{2.7}{1 + e} \times 1$$

$$e = 0.795$$

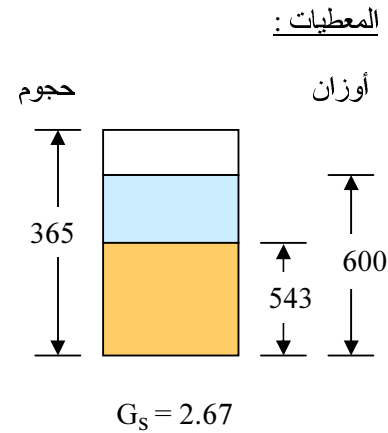
$$e \cdot S = G_s \cdot w$$

$$(0.795) S = 2.67(0.105)$$

$$S = 0.353 = 35.3 \%$$

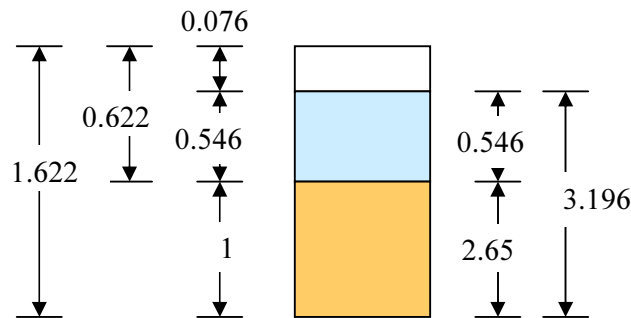
$$\gamma_{\text{sat.}} = \frac{G_s + e}{1 + e} \gamma_w$$

$$\gamma_{\text{sat.}} = \frac{2.67 + 0.795}{1 + 0.795} \times 1 = 1.93 \text{ gm/cm}^3$$



(2) The bulk density of a soil sample is 1.97 gm/cm^3 and its water content is 20.6 %. Taking the specific gravity 2.65, find the void ratio and degree of saturation. Draw the 3-phase diagram taking $V_s = 1 \text{ m}^3$. Check, the value of void ratio.

Solution:



(a)

$$\therefore \gamma_b = \gamma_d(1 + w)$$

$$1.97 = \gamma_d(1 + 0.206)$$

$$\gamma_d = 1.633 \text{ gm/cm}^3$$

$$\therefore \gamma_d = \frac{G_s}{1 + e} \gamma_w$$

$$1.633 = \frac{2.65}{1 + e} \times 1$$

$$e = 0.622$$

$$\therefore e \cdot S = G_s \cdot w$$

$$0.622 \times S = 2.65 \times 0.206$$

$$S = 0.877 = 87.7 \%$$

(b)

To complete the 3 - phase diagram:

$$V_s = 1 \text{ m}^3$$

$$W_s = 2.65 \text{ t}$$

$$W_w = 2.65 \times 0.206 = 0.546 \text{ t}$$

$$W = 2.65 \div 0.546 = 3.196 \text{ t}$$

$$V = 3.196 \div 1.97 = 1.622 \text{ m}^3$$

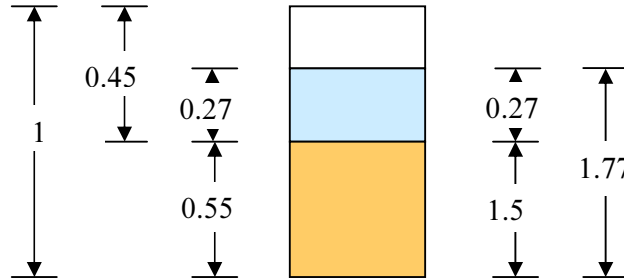
$$V_v = 1.622 - 1 = 0.622$$

$$V_w = 0.546 \text{ m}^3$$

$$V_a = 0.076 \text{ m}^3$$

$$e = \frac{0.622}{1} = 0.622 \text{ (O.K.)}$$

(3) The bulk and dry densities of a soil sample are 1.77 and 1.5 t/m³, respectively. If the degree of saturation is 60 %, what will be the specific gravity and porosity. Draw the 3-phase diagram taking $V = 1 \text{ m}^3$. Check the obtained values of specific gravity and porosity, then calculate the quantity of water added per unit volume for full saturation (without change of volume).



$$\because \gamma_b = \gamma_d (1 + w)$$

$$1.77 = 1.5 (1 + w)$$

$$w = 0.18$$

$$\because e \cdot S = G_s \cdot w$$

$$e \times 0.6 = G_s \times 0.18$$

$$\because \gamma_d = \frac{G_s}{1 + e} \gamma_w$$

$$1.5 = \frac{e \times \frac{0.6}{0.18}}{1 + e} \times 1$$

$$e = 0.818$$

$$G_s = 0.818 \left(\frac{0.6}{0.18} \right) = 2.73$$

$$n = \frac{e}{1 + e}$$

$$n = \frac{0.818}{1 + 0.818} = 0.45 = 45 \%$$

$$V = 1 \text{ m}^3$$

$$W = 1.77 \text{ t}$$

$$W_s = 1.50 \text{ t}$$

$$W_w = 1.77 - 1.50 = 0.27 \text{ t}$$

$$V_w = 0.27 \text{ m}^3$$

$$V_v = \frac{0.27}{0.6} = 0.45$$

$$V_s = 1 - 0.45 = 0.55 \text{ m}^3$$

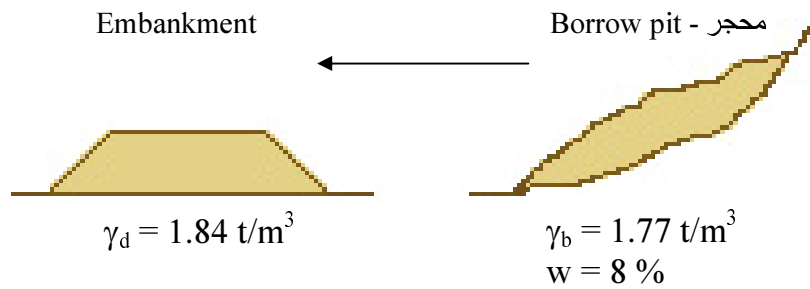
$$G_s = \frac{1.50}{0.55} = 2.73$$

$$n = \frac{0.45}{1} = 0.45 = 45 \%$$

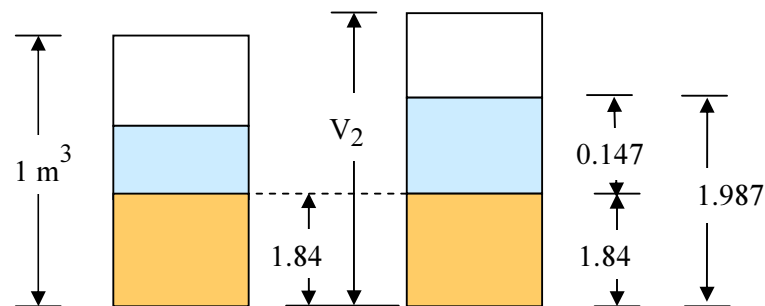
$$\Delta W_w = 0.45 - 0.27 = 0.18 \text{ t}$$

(4) An earth embankment is to be compacted to a dry density of 1.84 t/m^3 . The bulk density and water content of the borrow pit are 1.77 t/m^3 and 8 %, respectively. Calculate the volume of excavation in the borrow pit which corresponds to 1 m^3 of the embankment.

Solution:



In transporting soil from the borrow pits to the embankment, the weight of the solids remains unchanged. Solving the shown 3-phase diagrams for 1 m^3 of the embankment, we have:



For embankment :

$$V = 1 \text{ m}^3$$

$$W_s = 1.84 \text{ t}$$

For borrow pits :

$$W_s = 1.84 \text{ t}$$

$$W_w = 1.84 \times 0.08 = 0.147 \text{ t}$$

$$V_2 = 1.987 \div 1.77 = 1.12 \text{ m}^3$$

2.5 Problems

- (1) The porosity of a soil sample is 33 % and its specific gravity is 2.68. Calculate the void ratio, dry density, bulk density for 46 % degree of saturation, and saturated density. (Take water density = 1 gm/cm^3)
- (2) The bulk density of a soil sample is 1.85 t/m^3 , and its specific gravity is 2.7. Calculate the void ratio assuming that
- (a) The sample is dry
 - (b) The sample has a water content of 8.3 %
- (Take water density = 1 t/m^3)
- (3) A soil has a bulk density of 1.93 t/m^3 and water content of 44 %. Calculate the water content, if the density reduces to 1.76 t/m^3 without change of volume.
- (4) A soil sample has a void ratio of 0.81, degree of saturation of 84 % and specific gravity of 2.71. Calculate the bulk density and water content. What will be the water content at full saturation without change of volume. (Take water density = 1 gm/cm^3)
- (5) A soil sample has a dry density of 1.55 t/m^3 . The porosity and water content are 42 % and 25 %, respectively. Calculate the specific gravity and degree of saturation. (Take water density = 1 t/m^3)
- (6) The weight of a soil sample is 690 gm and its volume is 390 cm^3 . When 38 gm of water were added to the sample, without change of volume, its water content increased by 8 %. Calculate the saturated density and porosity. Take specific gravity 2.7. (Take water density = 1 gm/cm^3)
- (7) The weight of a soil sample is 320 gm and its volume is 210 cm^3 . The water content is 17 % and the specific gravity is 2.68. Draw the 3-phase diagram from which find volume of air voids, void ratio and degree of saturation.



